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Quantum Gravity via Random Triangulations of $\mathbf{R}^{(4)}$ and Gravitons as Goldstone Bosons of SL(4)/O(4)

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ABSTRACT

A model of random triangulations of a domain in $R^{(4)}$ is presented. The global symmetries of the model include SL(4) transformations and translations. If a stable microscopic scale exists for some range of parameters, the model should be in a translation invariant phase where SL(4) is spontaneously broken to O(4). In that phase, SL(4) Ward identities imply that the correlation length in the spin two channel of a symmetric tensor field is infinite. Consequently, it may be possible to identify the continuum limit of four dimensional Quantum Gravity with points inside that phase.

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1. Introduction and conclusions

Understanding four-dimensional Quantum Gravity ranks among the most important open problems in current theoretical physics. In the dynamical triangulation (DT) approach, one postulates that the metric degrees of freedom in d-dimensions can be represented by a discrete sum over abstract d-triangulations (for a recent review see ref. [1]). The DT approach is particularly successful in two dimensions [2]. Non-trivial scaling relations predicted by continuum techniques [3] have been confirmed in the DT model both analytically, by invoking its relation to matrix models, and by extensive numerical simulations.

The situation is different in the DT approach to four dimensional Quantum Gravity [4, 5]. Numerical simulations based on a discretized version of the Einstein-Hilbert action indicate the possibility of a phase transition between a "branched polymer" phase and a "crumpled" phase [4, 5, 6]. But, in the absence of analytical support for the numerical results, the latter should be taken with care. In particular, it is unclear whether or not the observed susceptibility peak signals a true phase transition, with critical couplings that remain finite as the infinite volume limit is taken [7, 8, 9].

General Relativity is a theory with a local spacetime symmetry group. It is well-known that general coordinate invariance is badly broken by any discretization of spacetime. The solution to this problem offered by the DT approach, is to pick a single representative for each microscopic metric in the form of an abstract triangulation. This solution may indeed be satisfactory in two dimensions. The reason, we believe, is related to the fact that two dimensional gravity is a trivial theory on the classical level. The dynamics of the quantum theory is dominated by strongly fluctuating metrics, and its physical states bare little resemblance to smooth two dimensional manifolds (or to smooth manifolds of any dimensionality, for that matter).

On the other hand it is an experimental fact that, in four dimensions, gravitational effects can be described accurately on a wide range of time and distance scales by a *classical* theory, that is to say, by General Relativity. One inevitably faces the difficult question of how to recover smooth macroscopical metrics out of the partition function of four dimensional DT [4, 9]. Since coordinate systems play a central role in the description of manifolds, an important problem is how to recover general covariance of the equations of motion on macroscopic scales. Considerable work has been devoted to these issues, but so far little progress was made.

What distinguishes four dimensional gravity from gravity in any lower dimension, is the existence of a massless spin two particle associated with the gravitational interaction – the graviton. One way to proceed is to ask what is the minimal subgroup of general coordinate transformations that is still potent enough to imply the existence of the graviton, and to try

to preserve only that smaller symmetry in the discretized model.

In a continuum framework, an answer to this question was given by Nakanishi and Ojima [10]. They observed that, instead of relating the masslessness of the graviton to general coordinate invariance, one can attribute it to the spontaneous breakdown of a large but *global* spacetime symmetry down to O(4).

The continuum theory of ref. [10] is of course a gauge fixed version of General Relativity, and the global symmetry of the gauge fixed lagrangian is GL(4). The order parameter for GL(4) symmetry breaking is the vacuum expectation value of the metric tensor $\langle g^{\mu\nu}\rangle \propto \delta^{\mu\nu}$. On a formal level, one can invoke the Goldstone theorem[1] to argue that there should exist massless excitations associated with the generators in the coset space GL(4)/O(4). We comment that the broken generators associated with physical spin two gravitons actually belong to SL(4)/O(4). The extra invariance under global scale transformations is therefore unnecessary for this reasoning.

General Relativity, or one of its extensions, are the only perturbatively consistent continuum theories that contain an interacting, massless spin two particle [13, 14]. Consequently, as long as we remain within the context of continuum field theory, considerations of the kind described above can only lead us to various gauge fixed versions of the (ill-defined) continuum path integral of General Relativity. But these considerations can nevertheless be helpful when we try to construct a discretized model that will reproduce General Relativity at sufficiently low energies.

In this paper we present a model of four dimensional Quantum Gravity whose global symmetries include SL(4) transformations and translations. The model is based on the ensemble of geometric triangulations of $R^{(4)}$, instead of the commonly used ensemble of abstract four-triangulations with a fixed topology. The finite volume partition function is defined by summing over random triangulations of a bounded domain $\mathcal{D} \subset R^{(4)}$. We assume that $\mathcal{D}$ is a closed, convex polyhedral domain.

Quantum Gravity requires some discretization of spacetime. This statement sounds so obvious, that one often forgets that it involves two distinct aspects of the theory. A non-perturbative construction requires the partition function's measure to be mathematically well-defined. The functional measure of the *continuum* path integral is ill-defined mathematically, and so some discretization of the partition function's measure is mandatory in any definition of Quantum Gravity.

A different question is whether spacetime itself is continuous or discrete at the smallest scale. In our model, every triangulation has a finite number of vertices, and the partition function's measure is mathematically well-defined. But every vertex is identified with a

[1] Earlier attempts to relate the masslessness of gauge particles, namely, the photon or the graviton, to the Goldstone theorem were made in refs. [11, 12].

point in $R^{(4)}$ and, in this sense, a *continuous four-dimensional spacetime* exists at the most fundamental level. This distinguishes the present model from both the DT approach [4, 5] and the quantum Regge calculus approach [15]. The results of this paper are due to the new measure, which depends explicitly on the coordinates of $R^{(4)}$. At this stage we have less understanding of the role of action, but it is unlikely that the commonly used Regge action will be the appropriate one.

Apart from the choice of boundary conditions, the definition of the partition function involves the *vector space* structure of $R^{(4)}$, but not its metric structure. As a result, the linear symmetry of the model is SL(4). This can be compared with (continuum or lattice) flat space field theories, where the linear symmetry is O(4) or a discrete subgroup of it. Now, while the global symmetry is enhanced to SL(4), the manifest symmetry of observables cannot exceed O(4). Hence, SL(4) has to break spontaneously. The usual metric structure of $R^{(4)}$ arises only after spontaneous symmetry breaking, and this fact is what allows *metric fluctuations* to have a dynamical role in the continuum limit.

The main subject of this paper is the derivation of SL(4) Ward identities. An obvious condition for obtaining a continuum behaviour, is that the length of links will remain finite as the infinite volume limit is taken. The precise condition is expressed in terms of suitable bounds on a probability distribution that characterizes the dominant link length. If this condition is satisfied, SL(4) is spontaneously broken to O(4), and the dominant link length can be used to define a stable, dynamical microscopic scale. We believe that this is also a sufficient condition for recovery of translation invariance in the infinite volume limit[2].

We will define below a positive definite symmetric tensor field $g^{\mu\nu}(x)$ whose value depends on the four-simplex containing the point x. As suggested by our notation, we propose to identify $g^{\mu\nu}(x)$ with the inverse metric tensor of General Relativity. The vacuum expectation value $\langle g^{\mu\nu}\rangle$ will be the order parameter for SL(4) symmetry breaking. In the SL(4)/O(4) broken phase, the SL(4) Ward identities imply that *the correlation length in the spin two channel of $g^{\mu\nu}(x)$ is infinite.* Thus, stability of the microscopic scale defined by the dominant link length is a necessary *and* sufficient condition for obtaining a non-trivial continuum behaviour.

Conventionally, a divergent correlation length is interpreted as signaling the presence of a massless particle. The validity of this interpretation depends on one's ability to carry out the analytic continuation to Minkowski space in a consistent way. Maison and Reeh [14] proved long ago that Goldstone bosons with non-zero spin cannot exist in a local, Lorentz covariant theory with a positive norm Hilbert space. Therefore, when we analytically continue back to Minkowski space, we should expect to obtain in the long distance limit a *gauge fixed* version

[2] A variant where translation and rotation invariance is further broken to a discrete subgroup could also lead to a consistent continuum limit, see the last section.

of General Relativity.

The implication of the Maison-Reeh theorem is that we are facing the familiar choice between a manifestly Lorentz covariant formulation and a manifestly unitary one. The model defined in this paper is manifestly covariant, but it lacks reflection positivity. In this formulation, the model cannot contain the physical graviton and be manifestly unitary at the same time. Hence, its validity depends ultimately on one's ability to construct a consistent set of physical observables. At the moment, the term "Goldstone boson" as used in this paper should be understood in a loose sense, namely, as implying only that the Fourier transform of the appropriate euclidean correlators has a zero momentum singularity.

2. The partition function

Each triangulation $\mathcal{T}$ of a domain $\mathcal{D} \subset R^{(4)}$ defines both an adjacency matrix (and, hence, an abstract four-triangulation with a boundary), and a division of $\mathcal{D}$ into simplexes. Let us denote the coordinates of the vertices of the triangulation by $y_i^\mu \in \mathcal{D}$, $i = 1, \ldots, N_0$. For $0 \le n \le 4$, the *geometrical* n-simplex $\Delta(y_{i_0}, \ldots, y_{i_n}) \subset \mathcal{D}$ associated with the *abstract* n-simplex $(i_0, \ldots, i_n)$ is defined to be the set of all points $x^\mu \in \mathcal{D}$ such that $x^\mu = \sum_{j=0}^n \lambda_j y_{i_j}^\mu$, where $\sum_{j=0}^n \lambda_j = 1$ and $\lambda_j \ge 0$. We demand that the union of all geometrical four-simplexes in $\mathcal{T}$ coincide with $\mathcal{D}$, and that if Δ_1 and Δ_2 are geometrical simplexes that belong to $\mathcal{T}$, then the same should be true for their intersection (if it is non-empty).

The definition of a geometric simplex in terms of the coordinates of its vertices makes use of the linear vector space structure of $R^{(4)}$. It does not depend on the $R^{(4)}$ norm. A similar statement applies to the volume of a geometric four-simplex $V = |\det W^\mu_{\ j}|$, where

$$W^\mu_{\ j}(y_0, \ldots, y_4) = y_0^\mu - y_j^\mu \,, \qquad \mu, j = 1, \ldots, 4\,. \tag{1}$$

We will allow the partition function's measure to depends only on $\det W$. As a result, the global symmetries of our model will include SL(4) transformations and translations.

The existence of an SL(4) symmetry will play a crucial role below. The choice of a domain in $R^{(4)}$ as the embedding space is dictated by the need to have a well-defined partition function, where the only SL(4) breaking effect is due to the boundary conditions. Other symmetric spaces such as a four-sphere or a four-torus are not vector spaces, and a knowledge of the adjacency matrix and the positions of the vertices is not sufficient to determine a unique division of the space. In the case of a four-sphere, the curvature induces an explicit SL(4) breaking effect locally. In the case of a four-torus, although locally it is indistinguishable from $R^{(4)}$, the possibility of an arbitrary winding number makes it unclear how to write down a well-defined partition function. We comment in passing that, in continuum gravity, one does not know how to define energy in the absence of spatial boundaries. (The electrostatic

analogy is that, by Gauss law, only an overall neutral charge distribution can fit into a cube with periodic boundary conditions). Consequently, one should expect difficulties with the recovery of stable single particle states, if one uses a four-sphere or a four-torus topology.

The partition function is defined as follows. The domain $\mathcal{D} \subset R^{(4)}$ will be taken to be a hypercube $-L/2 \leq x^\mu \leq L/2$ which we denote $\mathcal{D}_0$. A fixed three-triangulation is chosen on the faces of $\mathcal{D}_0$. We will define below an (inverse) metric tensor $g^{\mu\nu}(x)$. We assume that the boundary triangulation has a discrete rotation symmetry, which ensures that the expectation value of $g^{\mu\nu}(0)$ is proportional to the identity matrix.

The partition function is

$$Z = \sum_{\mathcal{T}} e^{-S(\mathcal{T})}\, \widetilde{Z}(\mathcal{T})\,, \tag{2}$$

$$\widetilde{Z}(\mathcal{T}) = \prod_{i=1}^{N_0} \int_{-L/2}^{L/2} d^4 y_i\, \Omega(y_1, \ldots, y_{N_0})\,. \tag{3}$$

The sum in eq. (2) runs over all abstract four-triangulations that have a realization in $\mathcal{D}_0$ with the prescribed boundary triangulation. It includes a summation over different identifications of the faces of the abstract triangulations with the faces of $\mathcal{D}_0$, and implicit in its definition is an appropriate symmetry factor needed to avoid double counting.

$S(\mathcal{T})$ is an action that depends only on the adjacency matrix of the abstract triangulation. The results of this paper depend mainly on the choice of the *measure* $\Omega(y_1, \ldots)$, and so the explicit form of the action will be left unspecified until the last section. Here we only comment that one can generalize $S(\mathcal{T})$ to include also matter fields.

$\widetilde{Z}(\mathcal{T})$ represents an integration over all realizations of the abstract triangulation $\mathcal{T}$ with a given identification of the boundaries. N_0 is the number of internal vertices of $\mathcal{T}$. The term realization is used to stress that we do not consider arbitrary embeddings of the vertices of $\mathcal{T}$ in $\mathcal{D}_0$. Allowed embeddings must preserve the abstract triangulation structure. Namely, if $(i_0, \ldots, i_k)$ is the intersection of two abstract simplexes $(i'_0, \ldots, i'_n)$ and $(i''_0, \ldots, i''_m)$, then the intersection of the corresponding geometrical simplexes $\Delta(i'_0, \ldots, i'_n)$ and $\Delta(i''_0, \ldots, i''_m)$ should be the geometric simplex $\Delta(i_0, \ldots, i_k)$.

In ref. [16], models of two dimensional DT embedded in some $R^{(n)}$ were considered, and it was found that area actions give rise to an instability against the formation of infinitely long spikes. As should be clear from the discussion of ref. [16], this instability does not occur if the dimensionality of the triangulation is equal to that of the target space, and one does not allow for arbitrary embeddings, but only for valid realizations of the abstract triangulation.

The restriction to valid realizations only, is enforced analytically as follows. Consider one realization of $\mathcal{T}$. Notice that this defines an orientation on $\mathcal{T}$. For every abstract four-simplex in $\mathcal{T}$, we now fix an ordering $(i_0, \ldots, i_4)$ such that $\det W(y_{i_0}, \ldots, y_{i_4}) > 0$. The

measure is defined as a product over all four-simplexes with the prescribed ordering

$$\Omega(y_1,\ldots,y_{N_0}) = \prod_{(i_0,\ldots,i_4)} F\left(\det W(y_{i_0},\ldots,y_{i_4})/\bar{a}^4\right)\,, \tag{4}$$

where[3]

$$F(s) = \begin{cases} s\,, & s \geq 0\,, \\ 0\,, & s \leq 0\,. \end{cases} \tag{5}$$

The vanishing of $F(s)$ for negative s, guarantees that two geometrical four-simplexes sharing a common tetrahedron will always lie on opposite sides of the hyper-surface defined by their common tetrahedron. Together with the abstract triangulation conditions fulfilled by $\mathcal{T}$, this guarantees that we have a legal realization. $\bar{a}$ is a reference microscopic scale defined below. We will usually set $\bar{a}=1$ and omit the $\bar{a}$-dependence. The explicit form of $F(s)$ eq. (5) favours four-simplexes with a large volume. However, as we discuss below, because of entropy considerations the actual volume distribution of four-simplexes should be exponentially damped.

Consider now the symmetric tensor $g^{\mu\nu}(x)$ defined as follows. We first define for each geometric four-simplex

$$g^{\mu\nu}(\Delta) = \sum_{<ij>} (y_i^\mu - y_j^\mu)(y_i^\nu - y_j^\nu)\,. \tag{6}$$

The sum is over the links of the four-simplex Δ. Notice that $g^{\mu\nu}(\Delta)$ is a strictly positive tensor for $\Omega(y_1,\ldots)\neq 0$. We now let $g^{\mu\nu}(x)=g^{\mu\nu}(\Delta)$ if x belongs to the interior of Δ. This defines $g^{\mu\nu}(x)$ except on a subspace of codimension one, which can be neglected under the integration in eq. (3).

A heuristic motivation to identify $g^{\mu\nu}(x)$ as defined above with the inverse metric tensor, comes from the following observation [17]. Let us add to $S(\mathcal{T})$ a scalar kinetic term, defined as a sum over all links of $(\phi_i-\phi_j)^2$. Suppose now that $\phi_i=\phi(y_i)$ for some function $\phi(x)$. If the approximation $\phi_i-\phi_j\approx(y_i^\mu-y_j^\mu)\partial_\mu\phi$ is a valid one, then the sum of $(\phi_i-\phi_j)^2$ over the links of a four-simplex takes the form of the standard continuum lagrangian for a scalar field in a gravitational background, where the role of the inverse metric tensor is played by $g^{\mu\nu}(\Delta)$.

The cubic symmetries of the boundary conditions imply $\langle g^{\mu\nu}(0)\rangle\propto\delta^{\mu\nu}$ in a finite volume. If translation invariance is recovered in the infinite volume limit, the same will be true for $\langle g^{\mu\nu}(x)\rangle = \langle g^{\mu\nu}(0)\rangle$. This would imply that the unique ground state is flat space. As we mentioned in the introduction, we believe that stability of the microscopic scale defined by the dominant link length, is a necessary and sufficient condition for recovery of translation and rotation invariance in the long distance limit.

[3] It should be possible to choose a measure such that the product $\prod d^4y_i\,\Omega(y_1,\ldots)$ will be GL(4) invariant. However, such a scale invariant measure will have a singular behaviour as the volume of any four-simplex tends to zero, and it is likely to yield undesirable instabilities.

$g^{\mu\nu}(0)$ will be our order parameter for SL(4) symmetry breaking. An SL(4) transformation acts on the coordinates of a vertex as $y_i^\mu \to (y_i')^\mu = A^\mu{}_\nu\, y_i^\nu$. Using eq. (6), $g^{\mu\nu}(0)$ is transformed into $(g')^{\mu\nu}(0) = A^\mu{}_\lambda\, A^\nu{}_\rho\, g^{\lambda\rho}(0)$. In an infinitesimal form, $\delta g(0) = B\, g(0) + g(0)\, B^T$, where $A = \exp(\alpha B)$. Therefore, the expectation value $\langle g^{\mu\nu}(0)\rangle$ breaks SL(4) down to O(4). The unbroken generators correspond to antisymmetric matrices, whereas the broken generators correspond to symmetric traceless matrices.

Before we turn to the derivation of the Ward identities, we wish to make one more comment on the dynamics of the model. For given values of parameters, the partition function should be dominated by triangulations with some typical number of vertices $\bar{N}_0$. Suppose now that we interchange the order of summation and integration in eqs. (2) and (3). We then start with a given set of $\bar{N}_0$ points, and we have to draw links connecting different pairs until the conditions of a geometric triangulation are ultimately fulfilled. A necessary condition for declaring that five given points are the vertices of a geometric four-simplex, is that the simplex will contain no other vertices of the triangulation. In the limit of large $\bar{N}_0$, the probability that this condition is satisfied will be proportional to $\exp(-V/\bar{a}^4)$ where $\bar{a}^4 = L^4/\bar{N}_0$.

The probability distribution for the volume of four-simplexes $\mathcal{P}(V)$ is actually a dynamical quantity. It depends not only on the entropy factor, but also on the explicit form of $F(s)$, as well as on the action $S(\mathcal{T})$. We expect its asymptotic behaviour to be $\mathcal{P}(V) \sim V \exp(-V/\bar{a}^4)$. This form should be valid at both small and large V, whereas at intermediate values, the role of the action $S(\mathcal{T})$ may lead to a more complicated behaviour.

The volume probability distribution, taken by itself, does not place any restrictions on the probability distribution for the length of links. The reason is that the volume of a four-simplex is SL(4)-invariant, whereas the length of a link is not. Here the boundary conditions play a crucial role. For given value of $\bar{a}$, we will choose the boundary triangulation such that boundary tetrahedra have a regular shape and their three-volume is $O(\bar{a}^3)$. As a result, the preferred link length near the boundary should be $O(\bar{a})$. Moreover, that tendency should prevail throughout the entire volume, as long as the average number of four-simplexes remains finite. The reason is that SL(4) is a *global* symmetry, and so the free energy should increase if we change the local average of an SL(4)-sensitive quantity such as the shape of four-simplexes.

Whether the stabilizing effect of the boundary conditions survives in the infinite volume limit, is the crucial dynamical question. The SL(4) Ward identities, to which we now turn, will clarify what are the dynamical conditions needed for obtaining a continuum behaviour in the infinite volume limit.

3. SL(4) Ward identities

A convenient basis for the generators in the coset space SL(4)/O(4) consists of six matrices $(B^{ij})^{\mu}{}_{\nu} = \delta^{i\mu}\delta^{j}{}_{\nu} + \delta^{j\mu}\delta^{i}{}_{\nu}$, $1 \le i < j \le 4$, together with three diagonal matrices $(B^{k})^{\mu}{}_{\nu} = \delta^{k\mu}\delta^{k}{}_{\nu} - \delta^{k+1,\mu}\,\delta^{k+1}_{\nu}$, $1 \le k \le 3$. For simplicity, we will assume below that B is either B^{12} or B^{1}. Notice that, considered as polarization tensors, these B-s lie in the (x^1, x^2)-plane, and they describe transverse spin two excitations if the only non-zero momentum components are p_3 and p_4.

The standard derivation of a Ward identity begins with the promotion of the constant parameter of the global transformation to a local one. We thus consider a *local* transformation of the form

$$y_i^{\mu} \to (y_i')^{\mu} = A(y_i)^{\mu}{}_{\nu}\, y_i^{\nu}\,, \qquad A(y) = e^{\alpha(y)B}\,. \tag{7}$$

We demand that this transformation be one-to-one, i.e. eq. (7) defines a *diffeomorphism*.

The requirement that $y'(y)$ be one-to-one places a non-trivial restriction on $\alpha(y)$. Unlike in any other case, we cannot assume that $\alpha(y)$ is a completely arbitrary function[4]. Let us consider two cases explicitly.

Case 1. We assume that $\alpha(x) = \alpha_0\, \theta_{\mathcal{D}'}(x)$, where $\theta_{\mathcal{D}'}(x) = 1$ if $x \in \mathcal{D}' \subset \mathcal{D}_0$, and $\theta_{\mathcal{D}'}(x) = 0$ otherwise. (This form should be viewed as a limiting case of a suitable family of diffeomorphisms). As mentioned above, we let $B = B^{12}$ or $B = B^{1}$. The mapping $y \to y'(y)$ will be one-to-one *iff* $\alpha(x)$ is a function of x^3 and x^4 only. For definiteness let us consider the slab $\mathcal{D}'(L')$ consisting of all points which satisfy $|x^3|, |x^4| \le L'$ for some $0 < L' < L$. We will take $\alpha(x) = \alpha_0\, \bar\theta(x)$, where $\bar\theta(x) = 1$ if $x \in \mathcal{D}'(L')$ and $\bar\theta(x) = 0$ otherwise.

The restriction to x^3- and x^4-dependence only, means that we can derive the Ward identity only for *transversal* polarization. A similar situation will be found below in the case of plane waves. Recall that under suitable dynamical conditions, the Ward identity will imply that the correlation length in the corresponding channel is infinite. It is encouraging that the Ward identity is applicable in the physically relevant case of a spin two channel. We comment, however, that the restriction to transversal polarization may be a technical one. It is plausible that the correlation length is infinite also in other channels of $g^{\mu\nu}(x)$ that correspond to the rest of the SL(4)/O(4) generators. The correlation length may ultimately be infinite in all channels of $g^{\mu\nu}(x)$ except for its trace.

For finite L, the transformation (7) with $\alpha(x) = \alpha_0\, \bar\theta(x)$ does not leave the boundaries in the x^1- and x^2-directions invariant. This should in principle give rise to an extra surface term in the Ward identity. As it turns out, that extra term vanishes identically, because $\Omega(y_1, \ldots)$ is zero whenever one of the internal vertices touches the boundary.

[4] As a result of this restriction, it is in general not possible to freely interchange an insertion of y_i^{μ} in the Ward identity with $\partial/\partial p_\mu$ is its Fourier transform.

Case 2. We assume that $\alpha(y) = \alpha_0 \,{}^{\sin}_{\cos}(py)$. To check when the transformation (7) is one-to-one, let us first calculate the jacobian $J = |\det \partial y'/\partial y|$ for general $\alpha(y)$. Using

$$\frac{\partial (y')^\mu}{\partial y^\nu} = A^\mu{}_\nu + \partial_\nu \alpha \, A^\mu{}_\rho \, B^\rho{}_\lambda \, y^\lambda \,, \tag{8}$$

we find

$$J = |1 + \partial_\mu \alpha \, B^\mu{}_\nu \, y^\nu| \;, \tag{9}$$

where we have used that for any two vectors u_i and v_j, $\det(\delta_{ij} + u_i v_j) = 1 + u_j v_j$.

Superficially, one can always force $y'(y)$ to be one-to-one simply by taking the parameter α_0 to be sufficiently small. But this is true only for finite L. In general, the limit $\alpha_0 \to 0$ involved in the derivation of the Ward identity and the limit $L \to \infty$ will not be interchangeable. Again, this difficulty is avoided in the case of spin two excitations. Requiring transversality, i.e. $p_\mu B^\mu{}_\nu = 0$, implies $J = 1$ identically.

To summarize, in both cases the transformation (7) is a *volume preserving* diffeomorphism (or a limiting case of such diffeomorphisms) which leaves the flat measure d^4y invariant.

Notice that the above transformations act on $\widetilde{Z}(\mathcal{T})$, and in this sense the Ward identity is derived triangulation by triangulation. One can ask why it is necessary to sum over an ensemble of triangulations in order to obtain a theory of gravity. While the following argument is certainly not rigorous, we believe that it captures the essential physics.

In order to see what might go wrong if we have a *fixed triangulation* model, imagine that we take one four-simplex and we blow it up. Namely, we move the vertices of that four-simplex to new positions, such that the length of every link in the new four-simplex is now huge compared to the average link length. To accommodate this change, we will have to empty from other vertices the entire volume of the new four-simplex. This amounts to a major deformation of the triangulation because, before that transformation, the volume now occupied by the single huge four-simplex typically contained many vertices. This means that the dynamics of a fixed-triangulation model should be non-local, *if the notion of locality is defined with respect to the $R^{(4)}$ norm.*

We comment that a "fixed triangulation model" can have a perfectly consistent continuum limit which, however, describes a relativistic field theory in *flat space*. One can take a periodic simplicial lattice as the fixed triangulation, and define a matter action on its sites. In this example, the dynamics of the matter fields is completely decoupled from the (non-local) dynamics of the y_i-s. The embedding $R^{(4)}$ plays no role in the continuum limit, and a consistent set of observables can be defined only in terms of the lattice sites. In the random triangulation model, on the other hand, operators that depend explicitly on the coordinates of $R^{(4)}$, like $g^{\mu\nu}(x)$, should play an important role in the continuum limit.

In the random triangulation model, if we take any finite number of vertices y_k and we move them to arbitrary new positions y'_k, then the most likely change will be relinking in the

vicinity of the old and the new positions of the vertices. The likelihood of a big change in any region which is far away from both the y_k-s and the y'_k-s should be negligible. Therefore, summing over an ensemble of triangulations can give rise to dynamics which is local with respect to the $R^{(4)}$ norm.

We are now ready to derive the SL(4) Ward identity for transversal polarization. Let us denote by Δ_0 the four-simplex containing the origin as an internal point. (Recall that cases where the origin is not an internal point of one four-simplex can be neglected). We begin by considering the following expectation value

$$G = Z^{-1} \sum_{\mathcal{T}} e^{-S(\mathcal{T})} \prod_{i=1}^{N_0} \int_{-L/2}^{L/2} d^4 y_i \, \Omega(y_1, \ldots, y_{N_0}) \, \mathrm{tr}\, B g(0) \,. \tag{10}$$

Taking $\alpha(y)$ to be as in one of the above two cases, the Ward identity is derived by making the change of variables (7) in eq. (10) and using the fact that $dG/d\alpha_0 = 0$ identically. Using the invariance of the flat measure under volume preserving diffeomorphisms, the Ward identity takes the form

$$\Big\langle \delta_{\alpha(y)} \, \mathrm{tr}\, B g(0) + \mathrm{tr}\, B g(0) \, \delta_{\alpha(y)} \log \Omega \Big\rangle = 0 \,. \tag{11}$$

Consider the first term. A straightforward substitution gives rise to

$$\Big\langle \delta_{\alpha(y)} \, g^{\mu\nu}(0) \Big\rangle = \Big\langle \sum_{<ij>\in\Delta_0} \Big(\delta_{\alpha(y_i)} \, y_i^\mu - \delta_{\alpha(y_j)} \, y_j^\mu \Big) \left(y_i^\nu - y_j^\nu \right) \Big\rangle + (\mu \leftrightarrow \nu) \,, \tag{12}$$

$$\delta_{\alpha(y_i)} \, y_i^\mu = \alpha(y_i) B^\mu{}_\lambda \, y_i^\lambda \,. \tag{13}$$

Notice that we cannot take $\alpha(y_i)$ outside the expectation value in eq. (12), because y_i is an integration variable. In other words, the function $\alpha(x)$ is promoted to an operator $\alpha(y_i)$.

Without being more specific, we will not be able to get any useful information out of the Ward identity. We therefore turn now to examine each of the above two cases separately. As we will see, one can obtain interesting physical results, provided a certain probability distribution that characterizes the dominant link length obeys suitable bounds.

Case 1. In this case we have (denoting $\delta_{\alpha(y)} \to \delta_1$)

$$\delta_1 \mathrm{tr}\, B g(0) = \mathrm{tr}\, B^2 (2q + r) \,, \tag{14}$$

where

$$q^{\mu\nu} = \sum{}' \, (y_i^\mu - y_j^\mu)(y_i^\nu - y_j^\nu) \,, \tag{15}$$

$$r^{\mu\nu} = \sum{}'' \, y_i^\mu (y_i^\nu - y_j^\nu) + (\mu \leftrightarrow \nu) \,. \tag{16}$$

Here $\sum'$ is a sum over those links of Δ_0 that belong to the interior of the slab $\mathcal{D}'(L')$. $\sum''$ is a sum over the links that intersect the boundaries of this slab, where the inner vertex is

y_i and the outer vertex is y_j. Notice that the q-term in eq. (14) is positive definite, whereas the r-term is not.

We now begin to see what are the dynamical conditions needed to obtain interesting physical conclusions from the Ward identity. What we need is that the dominant link length will define a stable microscopic scale. The r-term should then be negligible for L' which corresponds to some macroscopic scale.

In the forgoing discussion, it will be convenient to consider the probability distribution $\widetilde{\mathcal{P}}_x(l)$ for the length of the *longest link* of the four-simplex containing a given point x. Interesting continuum behaviour can arise only if the conditions stated below apply uniformly, and so we will usually omit the explicit reference to x. These conditions are as follows.
(a) The dynamical scale defined as $\tilde{a}^2 = \langle l^2 \rangle$, should satisfy $\tilde{a}/L \to 0$ in the infinite volume limit. Here the average is taken with respect to the probability distribution $\widetilde{\mathcal{P}}(l)$.
(b) In the limit $l/\tilde{a} \to \infty$, $\widetilde{\mathcal{P}}(l)$ should decrease faster than $(l/\tilde{a})^{-\beta}$ for some $\beta > 3$.

The definition of $\widetilde{\mathcal{P}}(l)$ allows us to focus on the relevant dynamical properties, and the scale $\tilde{a}$ characterizes the dominant link length. The behaviour of $\widetilde{\mathcal{P}}(l)$ depends not only on the distribution of link lengths, but also on the volume distribution of four-simplexes, because there is a higher probability of finding the point x inside a four-simplex with a large volume. One can think of the quantity $\Gamma(l) = -\log \widetilde{\mathcal{P}}(l)$ as an effective potential for the dominant link length. An interesting question is the relation between the "kinematical" microscopic scale $\bar{a}$ and the dynamical scale $\tilde{a}$. While a natural guess is $\bar{a} \approx \tilde{a}$, more complicated behaviour could arise because of the effect of the action $S(\mathcal{T})$.

If the above conditions are satisfied, we find for $L' \gg \tilde{a}$

$$\Big\langle \delta_1 \mathrm{tr}\, B g(0) \Big\rangle = C \,, \tag{17}$$

where

$$C \equiv \left\langle \sum_{<ij>\in\Delta_0} l_{ij}^2 \right\rangle . \tag{18}$$

Here $l_{ij}^2 = (y_i - y_j)^2$. It is clear from the above discussion that $C \approx \tilde{a}^2$. In deriving this result we have used the cubic symmetries and the fact that B^2 is a projection operator on the (x^1, x^2)-plane.

Turning now to the second term in eq. (11), we find

$$\delta_1 \log \Omega = \sum_{(i_0,\ldots,i_4)} \mathrm{tr}\, W^{-1}(y_{i_0},\ldots,y_{i_4})\, \delta_1 W(y_{i_0},\ldots,y_{i_4}) \,. \tag{19}$$

An explicit expression for $\delta_1 W$ is easily obtained using eqs. (1) and (13). One finds that $\mathrm{tr}\, W^{-1}\, \delta_1 W$ vanishes, except for those four-simplexes that intersect the boundary of the slab $\mathcal{D}'(L')$.

Putting the two terms together we obtain the Ward identity

$$-\left\langle \sum'' \mathrm{tr}\, W^{-1}\, \delta_1 W \ \mathrm{tr}\, Bg(0) \right\rangle = C\,. \tag{20}$$

As discussed above, eq. (20) is valid for $L' \gg \tilde{a}$, provided there exists $\beta > 3$ such that $\widetilde{\mathcal{P}}(l)$ is bounded by $(l/\tilde{a})^{-\beta}$ for $l \gg \tilde{a}$.

The Ward identity (20) takes the form of a surface sum over a two four-simplex correlator, where one four-simplex contains the origin, and the other intersects the boundary of the slab $\mathcal{D}'(L')$. Clearly, exponential damping of all correlation functions beyond some finite correlation length is incompatible with eq. (20). Consequently, if the above mild conditions on $\widetilde{\mathcal{P}}(l)$ hold, the Ward identity (20) implies that *the correlation length in the transversal channel of* $g^{\mu\nu}(x)$ *is infinite.*

Case 2. We next examine the content of the Ward identity in the case of plane waves. (We now let $\alpha(y) = \alpha_0 \exp(ipy)$ and denote the corresponding local variation by δ_2). Expanding $\exp(ipy)$ into a Taylor series and using eqs. (12) and (13) we find

$$\left\langle \delta_2 \mathrm{tr}\, Bg(0) \right\rangle = C + Rem. \tag{21}$$

The constant C, which represents the leading order contribution, is the same as in eq. (17). To estimate the remainder, we use the naive bound

$$|Rem.| \le \sum_{n=1}^{\infty} \frac{p^n \left\langle l^{n+2} \right\rangle}{n!}\,. \tag{22}$$

Here l stands for the length of the longest link of Δ_0.

A sufficient condition for neglecting the remainder in the limit $p\tilde{a} \to 0$, is $\widetilde{\mathcal{P}}(l) \sim \exp(-(l/\tilde{a})^\gamma)$ for some $\gamma > 1$. This condition can be rephrased by saying that the effective potential $\Gamma(l)$ should rise faster than linearly. In the case $\gamma < 1$ inequality (22) is useless, whereas the case $\gamma = 1$ is marginal, and convergence of the sum on the r.h.s. of inequality (22) depends on the subleading behaviour.

In the case $\gamma > 1$, or $\gamma = 1$ plus an appropriate subleading behaviour, one can also replace $\sin(p_\mu(y_i^\mu - y_j^\mu))$ by $p_\mu(y_i^\mu - y_j^\mu)$ in the second term $\delta_2 \log\Omega$. This give rise to the following Ward identity

$$p_\mu G^\mu(p) = C\,, \qquad (p\tilde{a})^2 \to 0\,, \tag{23}$$

where

$$G^\mu(p) = -i \left\langle \sum_{(i_0,\ldots,i_4)} e^{ipy_0} J^\mu(y_{i_0},\ldots,y_{i_4})\, \mathrm{tr}\, Bg(0) \right\rangle\,, \tag{24}$$

and

$$J^\mu = \sum_{j=1}^{4} (W^{-1})^j{}_\lambda B^\lambda{}_\nu W^\nu{}_j W^\mu{}_j\,. \tag{25}$$

In deriving this result we have used the transversality condition $p_\mu B^\mu{}_\nu = 0$ and the identity $\mathrm{tr}\, W^{-1}BW = 0$.

4. Discussion

The SL(4) Ward identities (20) and (23)–(25), together with the conditions for their validity, are the main result of this paper. The SL(4) Ward identities constitute the link between the dynamical properties of the discretized model and the desired continuum limit. What we need is that a stable microscopic scale, defined by the dominant link length, will exist in the infinite volume limit. If this condition is satisfied, we expect to obtain a translation invariant phase where SL(4) is broken down to O(4). The unique ground state of this phase should be flat space, and the existence of massless spin two excitations can be regarded as a consequence of the Goldstone theorem.

In this phase, the $R^{(4)}$ norm is a relevant concept of distance for low energy observables. A-priori, the only concept of distance in the DT approach is the microscopic geodesic distance, defined as the minimal number of links between two vertices. But if we succeed in building a phase with the above properties, then the $R^{(4)}$ norm should regain much of the physical significance that it has in ordinary field theories. An important property of a consistent continuum limit is that the low energy dynamics will be local with respect to the usual $R^{(4)}$ distance. As long as we stay sufficiently far away from matter concentrations, the $R^{(4)}$ distance should also provide a good first approximation to the *macroscopic geodesic distance* defined in terms of the expectation value of $g^{\mu\nu}(x)$ on some physical state. And the $R^{(4)}$ distance should coincide with the macroscopic geodesic distance on the ground state.

The dynamical scenario proposed in this paper is different from the one contemplated in previous works [4, 5]. We do not attempt to identify the continuum limit with a continuous phase transitions. Rather, the existence of spin two Goldstone bosons should characterize an *entire phase* of the model, and so every point inside that phase may be used to define the continuum limit[5].

If gravitons are Goldstone bosons, then it should be possible to derive the associated low energy theorems, and these low energy theorems should provide an opportunity to compare the predictions of the model with those of General Relativity. However, as we already explained in the introduction, carrying out this program involves very delicate issues. After analytic continuation to Minkowski space, the long distance limit of the correlation functions of the model should correspond to some gauge fixed version of General Relativity. In particular, we expect that the SL(4) currents will not be generally covariant in that limit.

[5] Recently it has been suggested that, in the DT approach, the continuum limit may be associated with regions inside the branched polymer phase [9].

As a result, uncovering the physical content of the low energy theorems, requires one to investigate what are the physical observables of the model. Understanding the properties of the *energy-momentum* tensor may play an important role in the construction of a consistent set of observables.

Apart from the Maison-Reeh theorem [14], there is another argument which indicates that the SL(4)/O(4) broken phase can only correspond to a gauge-fixed version of General Relativity. The diffeomorphism group is *locally non-compact*, and the local value of the metric tensor $g^{\mu\nu}(x)$ is an element of the coset space GL(4)/O(4). Even if we suppress the conformal degree of freedom, the corresponding coset space SL(4)/O(4), now considered as the space of values of $g^{\mu\nu}(x)$ with a fixed trace, is still non-compact. As a result, obtaining a manifestly diffeomorphism invariant version of General Relativity in the long distance limit, is incompatible with the finiteness of the expectation value of $g^{\mu\nu}(x)$. We comment that this behaviour might correspond in some formal sense to a "topological phase" characterized by uncontrolable stretching of simplexes.

An alternative line of investigation, is to look for a direct relation between the long distance limit of the euclidean correlation functions of the model and some gauge fixed version of *euclidean* gravity. To date, however, there is no widely agreed solution to the question of how the conformal factor should be treated in continuum euclidean gravity [18, 19]. This means that there is no generally agreed answer to the question of what is a consistent gauge fixed version of euclidean gravity. The dynamics of the present model singles out the trace of the metric tensor from the rest of its component. Thus, we hope that further investigations of the model may shed new light on this difficult issue.

One can also try to construct a manifestly unitary model, at the price of sacrificing manifest O(4) invariance [17]. To this end, one should first divide the hypercube $\mathcal{D}_0 \subset R^{(4)}$ into equal time slices. The rules for constructing triangulations on each time slice should be analogous to the ones used in this paper, and an extra rule has to be provided for the linking between neighbouring time slices. The global symmetries of this formulation should include discrete-time and continuous-space translations, as well as SL(3) transformations. The existence of physical spin two Goldstone bosons can then be attributed to the spontaneous breakdown of SL(3) to O(3). In this formulation one should prove the recovery of full Lorentz covariance in the low energy limit.

5. Dynamical considerations and the choice of the action

At this stage, we do not know whether the condition of faster-than-exponential damping of $\widetilde{\mathcal{P}}(l)$ can be satisfied for some range of parameters. (We also leave open the possibility that a weaker condition may be sufficient for the existence of momentum eigenstates). Numerical

simulations can clearly help us in getting some idea about the structure of the phase diagram, as well as about the behaviour of $\widetilde{\mathcal{P}}(l)$ in different phases. Here we only intend to give a very preliminary description of the issues in question, and to point out to possible relations between the choice of $S(\mathcal{T})$, the abstract triangulation's action, and the desired dynamics.

In principle, there may be two potential sources for an unsuppressed behaviour of $\widetilde{\mathcal{P}}(l)$ at large l. One possibility is that the shape of four-simplexes might fluctuate very strongly on small scales. Here the term "small scale" refers to the microscopic geodesic distance. In other words, we are asking what is the probability of finding a very elongated four-simplex (whose volume is $O(1)$) only a few links away from regular-looking four-simplexes.

Experimenting with geometrical triangulations suggests that big "short distance" fluctuations in the shape of simplexes should be strongly damped. We first observe that a very elongated four-simplex can never appear alone in a region of regular-shape four-simplexes. There must exist a transition region containing more and more elongated four-simplexes. If the length of the longest link in that region is $l_0 \gg 1$, then the typical number of four-simplexes in the transition region should be $O(l_0)$ or larger.

Moreover, consider two four-simplexes sharing a common tetrahedron, and assume that one of them is much more stretched than the other. Namely, assume that the longest link of one of them is longer than the longest link of the other by a large factor $z \gg 1$. Under these circumstances, the phase space available for the common vertices will typically be reduced by a factor of $O(1/z)$. The origin of this reduction is the need to keep the volume of *both* four-simplexes $O(1)$. If we assume even a finite suppression factor for every pair of four-simplexes in the transition region, we arrive at an overall exponential suppression factor as a function of l_0. Needless to say, it will take a more serious investigation to determine what is the actual behaviour of $\widetilde{\mathcal{P}}(l)$. But one can at least hope that the condition of faster-than-exponential damping is not totally unreasonable.

There is another potential source for an undamped behaviour of $\widetilde{\mathcal{P}}(l)$, which is related to the long range dynamics. Let us assume that strong short distance fluctuations in the length of links are indeed suppressed. Thus, when we start near the boundary, we should find mainly regular-shape simplexes, and the quasi-local average of every entry of $g^{\mu\nu}$ should be $O(1)$.

However, the local SL(4) order parameter may gradually change as we move away from the boundary. What is the probability that we will ultimately reach regions where *all* four-simplexes are extremely stretched in one direction and extremely squeezed in another direction? If this happens, then we are facing an infra-red instability, and $\widetilde{\mathcal{P}}(l)$ will show no damping at all at large l. In this case we can expect no more than the trivial constraint $\widetilde{\mathcal{P}}(l) = 0$ for $l > 2L$.

The answer to the above question may depend critically on the dimensionality of the

model. What we have described above is nothing but the familiar mechanism for the restoration of continuous symmetries in *two dimensions*. It is plausible that the same physical mechanism should work in a two dimensional version of the present model as well.

How can this be reconciled with our previous argument that the expectation value of $g^{\mu\nu}$ necessarily break SL(d) down to O(d)? The resolution of this paradox is that a *well-defined* SL(d)-*symmetric phase does not exist*. The reason is that, as mentioned earlier, the coset space SL(d)/O(d) is non-compact. Consequently, a presumed SL(d)-symmetric phase would have to be dominated by infinitely stretched simplexes.

A two dimensional version of the present model may be unstable against long range variations in the local SL(2) order parameter, as in the case of any other continuous symmetry in two dimensions. If this is indeed the case, then, away from the boundary, all triangulations will be dominated by extremely stretched triangles, whose length will be limited only by the finite size of the embedding square. As the infinite volume limit is taken, one may obtain a phase where most of the triangles have a finite distance from the boundary. That "topological" phase will be neither translationally invariant nor SL(2)-symmetric[6].

The infra-red instability described above is known to be a peculiar property of two dimensional kinematics. There is no obvious reason to suspect that our *four dimensional* model will suffer from a similar instability. But, going to four dimensions does not automatically guarantee the absence of an infra-red instability. The latter could occur if for some reason the would-be Goldstone modes developed a $1/p^4$ behaviour instead of the usual $1/p^2$ pole. The issue will have to be investigated in the future, alongside with the effect of short distance fluctuations on $\widetilde{\mathcal{P}}(l)$.

How do these considerations affect the choice of the action $S(\mathcal{T})$? In our model, the metric tensor $g^{\mu\nu}(x)$ is formally a composite field, and there is no obvious advantage in using the Regge discretization of the Einstein-Hilbert action. (By analogy, we do not expect to find an explicit kinetic term for pions in the QCD lagrangian). Since we will not use the Regge action, we also avoid the unboundedness problem that may aflict that action in the infinite volume limit.

A possible choice of the action is

$$S(\mathcal{T}) = \kappa_4 N_4 + \gamma' \sum_i \left(\mathcal{N}_4(i) - \bar{\mathcal{N}}_4\right)^2 \ . \tag{26}$$

Here $\mathcal{N}_4(i)$ denotes the number of four-simplexes sharing the i-th vertex, and $\bar{\mathcal{N}}_4$ is some average value of $\mathcal{N}_4(i)$. As usual, N_4 is the total number of four-simplexes. The first term on the r.h.s. of eq. (26) is the familiar cosmological term, whereas the second term suppresses the occurrence of high $\mathcal{N}_4$ vertices. The action (26) is positive definite for $\kappa_4, \gamma' \geq 0$. Notice

[6] If this behaviour is verified, it would imply that a two dimensional version of the present model is no exception to the "$c = 1$ barrier" [20].

that the action (26) is local. On the other hand, the term $(N_4-V)^2$ which is commonly added to the Regge action in numerical simulations is not, because one cannot write $(N_4-V)^2$ as a single sum over a local quantity.

The grand-canonical partition function is well-defined only if the number of triangulations is exponentially bounded as a function of N_4. At present, it is unclear whether or not the exponential bound exists in the ensemble of abstract four-triangulations with a fixed topology [7, 8, 9]. The restriction to geometrical triangulations may carry the extra benefit of allowing us to establish an exponential bound. The infinite volume limit should then be defined by letting $\kappa_4 \to \kappa_4^c$ from above. (Otherwise, one would have to use the canonical partition function. In that case it is convenient to take N_0 as the independent variable, and to define the infinite volume limit by letting $N_0 \to \infty$).

We propose to include the second term in the action for the following reason. We believe that there is a correlation between strong short distance fluctuations in the SL(4) order parameter, and the occurrence of high $\mathcal{N}_4$ vertices. The tendency for the buildup of a high $\mathcal{N}_4$ increases as we try to decrease the size of the transition region between an extremely long four-simplex and a surrounding of regular four-simplexes. The increased likelihood for high $\mathcal{N}_4$ vertices in the transition region, was taken into account in our previous argument concerning the expected suppression of short distance fluctuations in the SL(4) order parameter. But if the resulting large-l damping of $\widetilde{\mathcal{P}}(l)$ turns out to be marginal, then the extra necessary damping may be provided by the second term in eq. (26).

A related issue is the role of *crumpled* triangulations in the present model[7]. Crumpled triangulations are characterized by the existence of vertices with very high coordination number (or, equivalently, very high $\mathcal{N}_4$). The high connectivity may prevent many abstract crumpled triangulations from having a realization in $R^{(4)}$. In particular, the occurrence of vertices whose $\mathcal{N}_4$ is of the same order of magnitude as the total number of four-simplexes should be suppressed. Moreover, if an abstract crumpled triangulation does have a realization in $R^{(4)}$, then every high $\mathcal{N}_4$ vertex, as well as every vertex which is a neighbour of a high $\mathcal{N}_4$ vertex, should typically have a very limited room to move around. We thus expect that the contribution of crumpled triangulations to the partition function will be suppressed in the present model from the outset.

Apart from the parameters that enter the action eq. (26), the model contains an additional free parameter. This parameter is the power of s in eq. (5). The results of this paper remain valid in the more general case $F(s) = \theta(s)\, s^n$ for $n > 1$, with the minor change of a factor of n in the appropriate places (e.g. the r.h.s. of eq. (19)). The role of the parameter n may resemble the role of the inverse temperature in classical statistical mechanics. If true,

[7] There is some evidence [9, 8] that, in the ensemble of abstract triangulations with a four-sphere topology, crumpled triangulation overwhelm in the limit $N_4 \to \infty$.

then this parameter may have a significant effect on the dynamics of the model.

In this paper we made no attempt to carry out a systematic investigation of the phase diagram of the model. We focused on the physical properties that should characterize a translation invariant phase where SL(4) is spontaneously broken to O(4). Another phase having a stable microscopic scale may be characterized by further breaking of rotation and translation invariance to a discrete subgroup. This would be the case if the ground state resembles a regular lattice. Although we expect a different (and richer) massless spectrum is that phase, it may also give rise to a physically interesting continuum limit. Another possibility is that of a "topological phase" characterized by a divergent link length in the infinite volume limit. Future investigations should help us in getting a more detailed understanding of the model and its phase diagram, and in deciding whether the dynamical scenario proposed in this paper is viable.

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